Secure the Biscay bay from intruders with a group of underwater robots

L. Jaulin and B. Zerr

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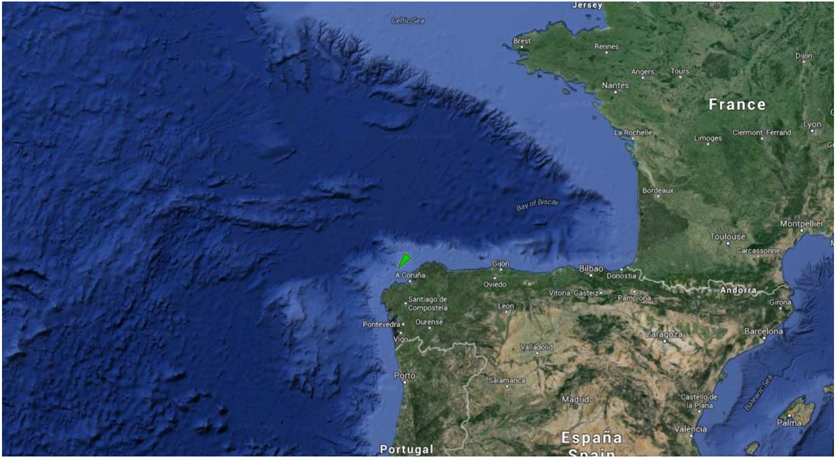
Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne

Le navire a été repéré en janvier. Ce serait la première fois depuis la fin de la Guerre Froide qu'un tel sous-marin, doté de missiles nucléaires, se serait aventuré dans cette zone au large des côtes françaises.



Bay of Biscay 220 000 km^2

An intruder

- Several robots $\mathscr{R}_1,\dots,\mathscr{R}_n$ at positions $\mathbf{a}_1,\dots,\mathbf{a}_n$ are moving in the ocean.
- If the intruder is in the visibility zone of one robot, it is detected.

Complementary approach

- We assume that a virtual intruder exists inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}(t)).$$

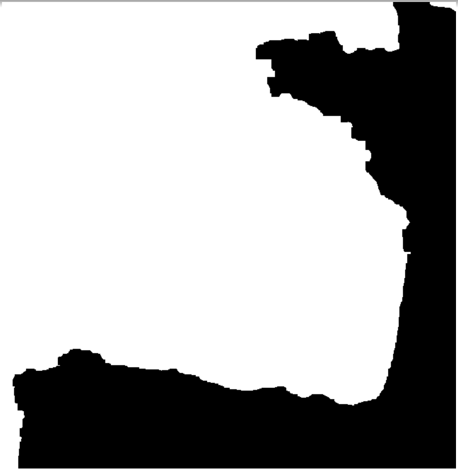
- Each robot $\mathscr{R}_i$ has the visibility zone $g_{\mathbf{a}_i}^{-1}([0, d_i])$ where d_i is the scope.

Theorem. An (undetected) intruder has a state vector $\mathbf{x}(t)$ inside the set

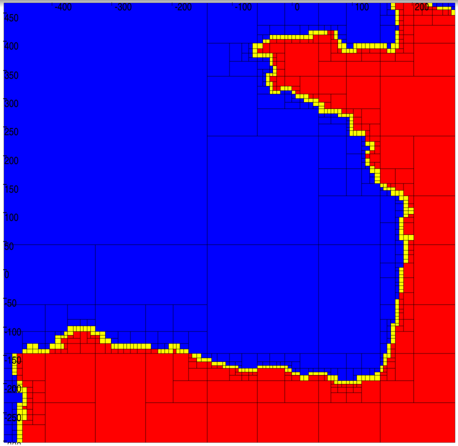
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

$$\mathbb{S}(t) = \overline{proj_{world}(\mathbb{X}(t))}.$$



Set $\mathbb{G}$ in white


-400
-300
-200
-100
0
100
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450
400
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300
250
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50
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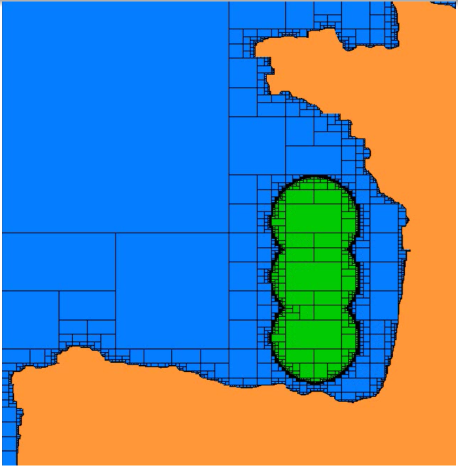
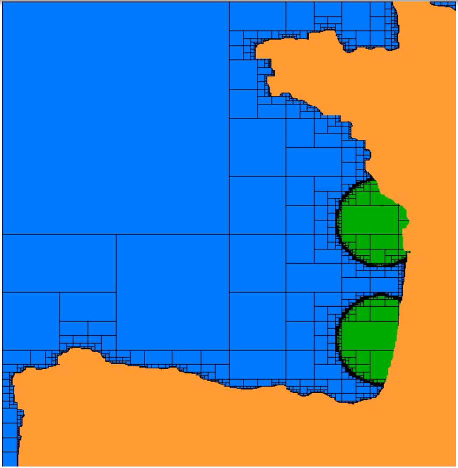
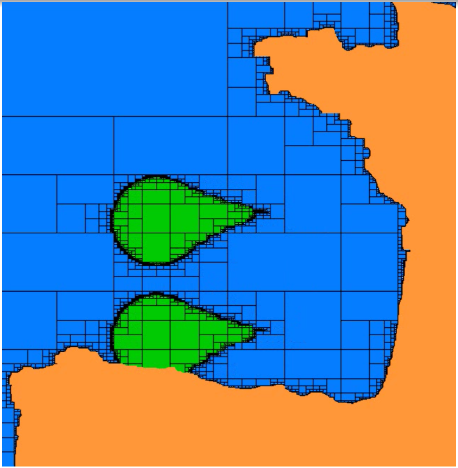
-50
-100
-150
-200
-250



green: $\bigcup_i g^{-1}_{\mathbf{a}_i(t)}([0, d_i(t)])$

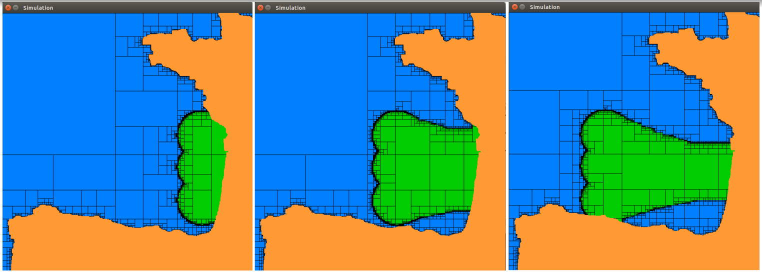


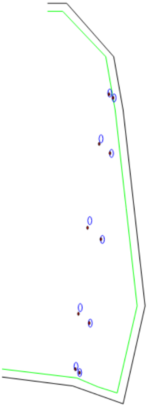
Blue: $\mathbb{G} \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty])$



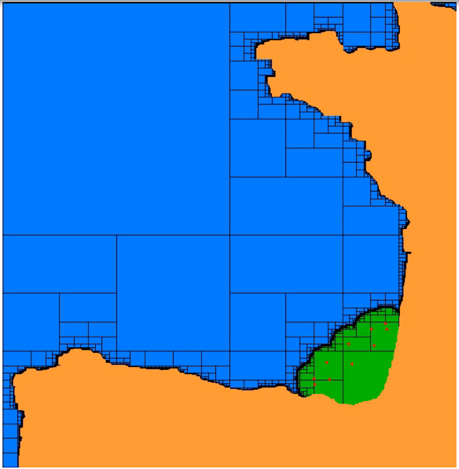
Blue:

$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty]).$$


Simulation
Simulation
Simulation

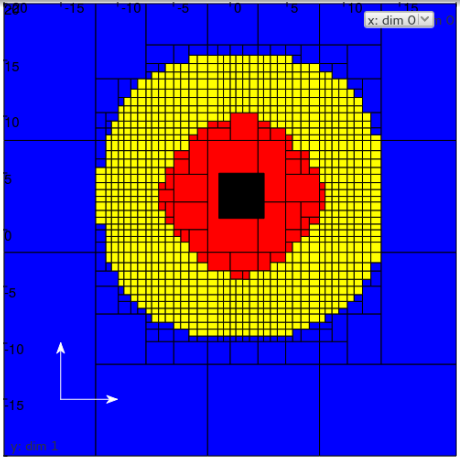


Strategy of the ellipse



Video : https://youtu.be/rNcDW6npLfE

Thick sets


-15
-10
-5
0
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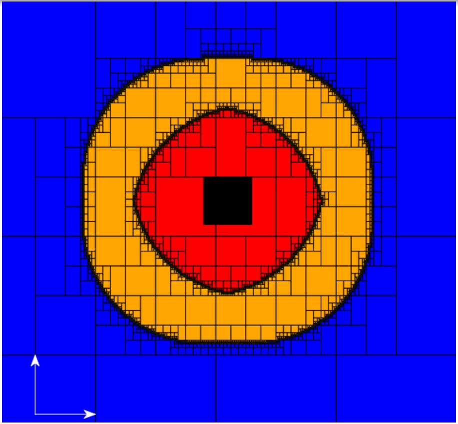
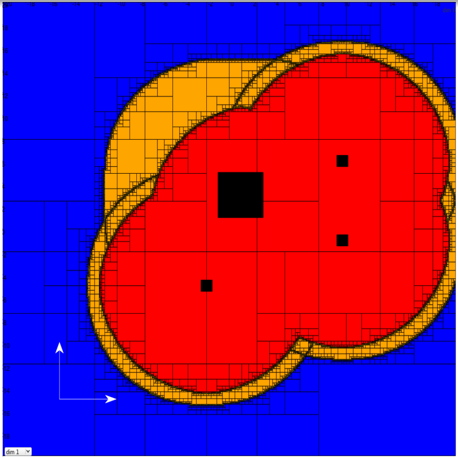
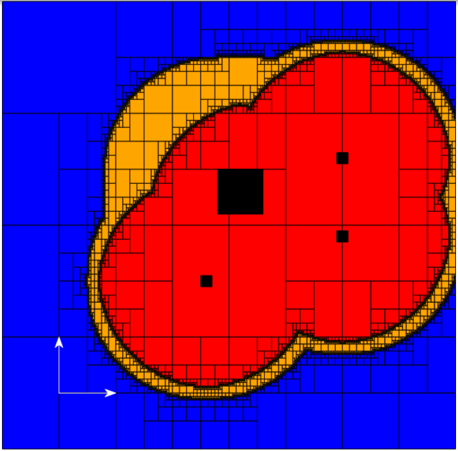
x: dim 0
15
10
5
0
-5
-10
-15
y: dim 1







If $\mathbf{a}_i(t) \in [\mathbf{a}_i](t)$, the thick observer is

$$[\![\mathbb{X}]\!](t) = \mathbb{G} \cap ([\![\mathbb{X}]\!](t-dt) + dt \cdot \mathbb{F}([\![\mathbb{X}]\!](t-dt))) \cap \bigcap_i g^{-1}_{[\mathbf{a}_i](t)}([\![[d_i(t)], \infty]\!]).$$

Non causal secure zone

Idea: Take into account the future.

The feasible set can be obtained by the following contractions

$$\begin{array}{rcl} \overrightarrow{\mathbb{X}}(t) & = & \overrightarrow{\mathbb{X}}(t)\cap(\mathbb{X}(t-dt)+dt\cdot\mathbb{F}(\mathbb{X}(t-dt))) \\ \overleftarrow{\mathbb{X}}(t) & = & \overleftarrow{\mathbb{X}}(t)\cap(\mathbb{X}(t+dt)-dt\cdot\mathbb{F}(\mathbb{X}(t+dt))) \\ \mathbb{X}(t) & = & \overrightarrow{\mathbb{X}}(t)\cap\overleftarrow{\mathbb{X}}(t) \end{array}$$

with the initialization

$$\mathbb{X}(t)=\overrightarrow{\mathbb{X}}(t)=\overleftarrow{\mathbb{X}}(t)=\mathbb{G}.$$